Viscous-Inviscid Interactive Procedure for Rotational Flow in Cascades of Airfoils

W. Johnston*
Georgia Institute of Technology, Atlanta, Georgia
and
P. Sockol†
NASA Lewis Research Center, Cleveland, Ohio

Abstract

A VISCOUS-INVISCID interactive calculation procedure for application to flow in cascades of two-dimensional airfoils is described. This procedure has four components. The first is a grid generation method that relies in part on a succession of conformal mappings to produce a nonorthogonal curvilinear grid mesh fitted to the geometry of the cascade. This method can accommodate an arbitrarily specified cascade geometry. Second, a numerical solution of the Euler equations is carried out on this grid. Third, a viscous solution for use in boundary-layer and wake regions has been programmed. Finally, an interactive scheme which takes the form of a source-sink distribution along the blade surface and wake centerline is employed.

Contents

In recent years, a great deal of progress has been made in the development of faster, more efficient numerical procedures for the calculation of flow past aerodynamic shapes. Algorithms for the solution of the Euler equations and Navier-Stokes equations have been available for some time; for example, MacCormack's method,[1] an explicit time-marching procedure, has been widely used since its introduction in 1969. More recently, time-marching algorithms that have an implicit character[2-4] have been introduced. In addition to the need for an efficient flow calculation algorithm, another requirement in aerodynamic calculations is some technique for dealing with the complex geometries that often occur. Several different grid generation schemes have been developed in recent years to meet this requirement. Certainly among the most popular of these is a versatile method for dealing with aerodynamic geometries developed by Thompson, Thames, and Mastin.[5] Steger[6] has combined the Beam and Warming implicit finite difference algorithm[4] with the grid generation procedure of Thompson et al.[5] to simulate compressible flow about isolated airfoils.

While similar in many respects to flow calculations for isolated airfoils, flow calculations for cascades encounter some additional difficulties in terms of the geometry and the boundary conditions that must be applied. The necessity for dealing effectively with complicated geometries in cascade flow problems has led to the development of several diverse geometry procedures. Recently, Steger et al.[7] have applied the approach used by Steger in the isolated airfoil problem to flow through cascades. In the present research effort, a body-fitted nearly orthogonal curvilinear grid is generated by a method relying in part on a succession of conformal mappings. An implicit time-marching solution of the Euler equations is then carried out on this grid in the manner described in Refs. 6 and 7 except for certain differences in the treatment of boundary conditions. We have accounted for the effect of viscosity on the flow by coupling the inviscid calculation with a separate viscous shear layer calculation in an interactive procedure. The various components of this viscous-inviscid interactive calculation are now discussed separately; additional details may be obtained by consulting Ref. 8.

The inviscid computations of the present work are performed on a C-type body-fitted grid in which one family of lines form open loops (Cs) around the blade and wake. The grid is periodic and nearly orthogonal. This choice permits accurate resolution of the leading-edge region and provides an appropriate location for the interactive wake boundary conditions. Typical grids for a turbine cascade and a compressor cascade are shown in Figs. 1 and 2. The grid generation employs two analytical mappings that take the multiply connected exterior of a cascade of airfoils to the interior of a simply connected domain. A numerically constructed mapping is then used to take this region into a rectangular computational space. During this process, a small amount of coordinate straining is introduced in the vicinity of the blade trailing edge to insure grid continuity across the wake. Consequently, this grid continuity is obtained at the expense of a small amount of nonorthogonality.

The inviscid component of this interactive procedure consists of a time-marching solution of the Euler equations using an approach described in detail in Ref. 6. The successful application of this method to cascade flows, however, seems to require a more careful treatment of certain computational boundaries than is generally the case with isolated airfoils.

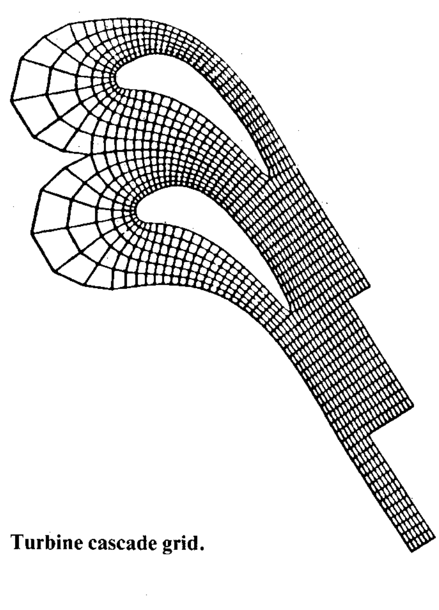

Fig. 1 Turbine cascade grid.

Presented as Paper 83-0256 at the AIAA 21st Aerospace Sciences Meeting, Reno, Nev., Jan. 1983; submitted March 7, 1983; synoptic received Aug. 5, 1983. Full paper available from AIAA Library, 555 W. 57th Street, New York, N.Y., 10019. Price: Microfiche, $4.00. **Remittance must accompany order.**

*Assistant Professor, School of Engineering Science.
†Head, Viscous Methods Section.

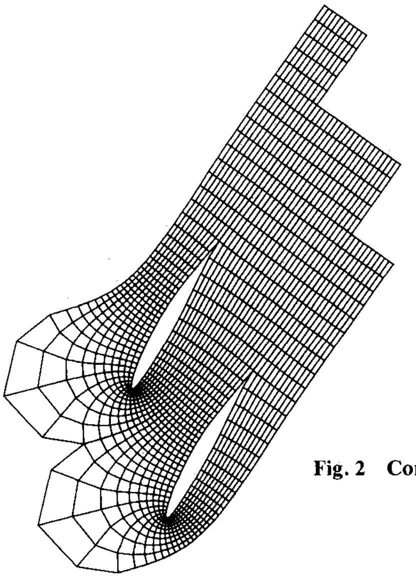

Fig. 2 Compressor cascade grid.

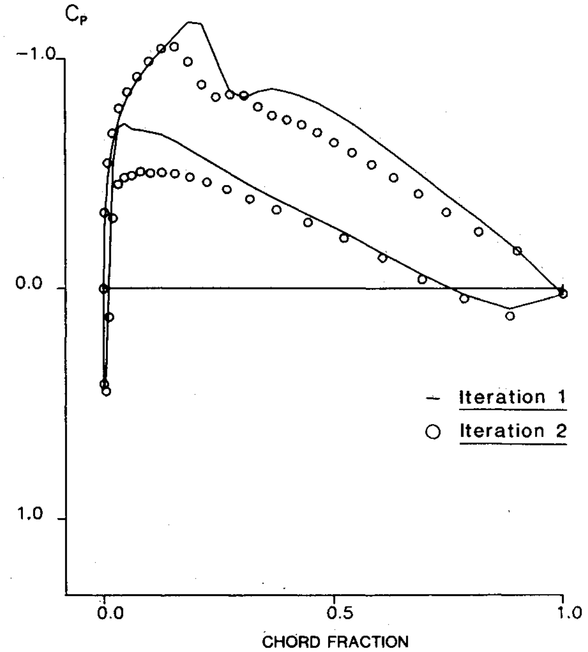

Fig. 3 Surface pressure coefficient (NACA 65-410 cascade, $M_\infty = 0.83$, 7.3 deg angle of attack).

More specifically, we have used a characteristics treatment at the upstream boundary and a nonreflecting downstream boundary condition based on the work of Rudy and Strikwerda.[9] It is important to note that these boundary procedures were employed only after simpler approaches had failed. For example, the Rudy and Strikwerda approach to the downstream boundary was adopted after the method of specifying the exit pressure and extrapolating other dependent variables was found to cause the solution to become unstable. While such procedures are often employed in isolated airfoil calculations to assist with the convergence rate of the solution, it was our experience that in the absence of such precautions the cascade solution either converged to a result with noticeable errors or did not converge at all.

The viscous solution consists of a finite difference calculation that is capable of dealing with blade boundary layers and the wake. The calculation can accommodate a flow that is compressible and turbulent; also, some transition modeling has been incorporated within the calculation. The interactive procedure consists of an iteration between the inviscid and viscous solutions previously described. The effect of the presence of the viscous shear layer on the inviscid solution is modeled as a source-sink distribution along the blade surface and wake centerline. Results have been obtained with this numerical procedure for several cascade flow situations. A more detailed description of these results may be found in Ref. 8; here it will suffice to mention that the method has performed well in a number of cascade flow situations. Furthermore, certain calculations were performed that clearly indicated the need for some viscous corrections to the inviscid results. For example, a transonic flow through a 45 deg staggered cascade of NACA 65-410 blades with a gap-chord ratio of 0.78, shown in Fig. 2, produced the blade pressure coefficient distributions found in Fig. 3. A comparison of these pressure coefficient plots, taken from first and second inviscid solutions within the viscous-inviscid iterative procedure, demonstrates this fact. It should be mentioned that the method in its present form has some limitations, most notably in its applicability to flows with boundary-layer separation, but it would appear that with some modification the method could be improved in this regard. While a Navier-Stokes approach to the inclusion of viscous effects within the numerical calculation would undoubtedly be required in certain severe flow situations (e.g., massive boundary-layer separation with vortex shedding), the viscous-inviscid interactive approach provides an alternative in the analysis of less severe flow situations such as a cascade operating at or near design conditions. In a time-marching solution of the Navier-Stokes equations, it is unlikely that the computational effort expended per grid point will greatly exceed the value for a time-marching solution of the Euler equations. However, it is an advantage of the interactive approach that this inviscid time-marching solution is carried out on a grid that is sparse in comparison with the grid requirements of a Navier-Stokes solution.

Acknowledgment

The first author acknowledges NASA Lewis Research Center for the support they provided under Grant NSG-3260.

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